Resistance and Radiation of Tungsten as a Function of Temperature

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IN a paper[1] by one of the present authors and Dr. Worthing, data on the radiating characteristics of tungsten obtained as a result of study that extended over a period of about ten years prior to 1922 were given. Some data published[2] since, indicated that there were some discrepancies in the temperature-resistance data given in that report. To check this, the Lamp Department Wire Division furnished some very pure 10-mil tungsten wire that had been very carefully drawn through tested dies so as to produce wire that was as uniform and as nearly round as possible. Mr. P. P. Tarasov, metallurgist at the Wire Division, made some tests of this wire both before and after it was used in our experiment and has the following to say concerning it:

> Structurally this represents a typical non-sag tungsten wire. As viewed on a microscope, its structure consists of very long grains occupying, in places, the whole transverse section of the wire. Occasional small grains are found "imbedded" within the large grains or at grain boundaries.
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> At high magnifications the metal appears quite homogeneous and free from inclusions, so that, if it contains any impurities or admixtures, such as carbon, iron, silicon, potassium, etc., they must be present there in minute quantities and in a state of extreme dispersion or in solid solution with tungsten.
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> The surface of the wire bears many grooves which apparently are "drawing marks." A grain boundary with few small grains can also be observed on these photomicrographs.
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> The measurement of the diameter of the wire with a micrometer checked by measurements of photomicrographs of a cross section showed that the wire is slightly (less than two percent) out-of-round.

The temperatures given in this paper are on the scale that has been used at the Nela Park Laboratories for the past ten years. On this scale the gold point is 1336°K and C_2 is taken 14,330μ°. Previous work[3] shows that temperature measurements made at Nela Park agree well with like measurements made at the various Standardization Laboratories. Worthing's average values[1] for the emissivity of tungsten were used to reduce the brightness temperature obtained with the optical pyrometer to true temperatures. While the wire used in this experiment had evidences of die marks it had a very good polish which was probably quite similar to that of the filaments used in previous experiments.

Several lamps with long hairpin filaments having potential leads, were made of samples of this wire and the current and voltage measured for temperatures between 1200 and 2800°K. The logarithm of the resistance was plotted against the logarithm of the temperature and well within the limit of the measurements, the relation was found to be linear with a slope of 1.195 which agrees very well with the average of the previous result[1] of 1.20 for the slope of this logarithmic relation. It might be added here that no indication of anything except a linear relation for the logarithm of the resistance plotted against the logarithm of the temperature for a range between 1200 and 3000°K has ever been found in this laboratory. This statement is based on measurements of very many lamps and very many samples of tungsten wire. Since one part of this test was to study the characteristics of tungsten for temperatures between 1200°K and room temperature it was necessary either to eliminate the effect of losses[4] due to the heat conducted away by the leads or in some manner to make corrections for the effect of these losses.

Calculations similar to those given in a paper by Langmuir[5] and associates show that for a

[1] W. E. Forsythe and A. G. Worthing, Astrophys. J. **61**, 146 (1925).

[2] Zwikker, Physica **5**, 249 (1925); Jones, Phys. Rev. **28**, 202 (1926); Geiss, Ann. d. Physik **79**, 85 (1926).

[3] W. E. Forsythe, Phys. Rev. **38**, 1247 (1931).

[4] Langmuir, MacLane and Blodgett, Phys. Rev. **35**, 478 (1930); E. P. Hyde, F. E. Cady, A. G. Worthing, Trans. Illum. Eng. Soc. **6**, 238 (1911).

[5] Langmuir, reference 4.

10-mil filament (0.025 cm in diameter) of tungsten wire an appreciable change of temperature due to the end conduction does not extend further than about 40 cm for a temperature of 450°K. Accordingly, to extend the test to very low temperatures a lamp was made up of this selected wire with a hairpin filament with an overall length of the filament of about 110 cm with potential leads attached across the lower 25 cm of the filament. All of the metal inside of the lamp that constituted the potential leads was tungsten and all joints were carefully welded. The part of the potential leads attached to the side of the large filament were coiled filaments made of wire about 0.02 mm in diameter. The amount of heat conducted away by wire of this size is very small. This lamp was first made up gas-filled and aged for about two hours at 3000°K. Then in evacuating the bulb the pumps were kept running several hours with the filament heated to about 1000°K and the lamp heated in a furnace to about 350°C. Attached to the side of this lamp was a tube containing charcoal. During the evacuation of the lamp this charcoal was heated by a specially attached furnace to about 400°C for about an hour. First, the temperature, current and voltage of this lamp were measured for a range of temperatures from 1100°K to 2800°K. This was repeated four times, half of the readings being taken when the temperature was increased and the other half when the temperature was decreased, the filament being allowed to operate at each current for fifteen minutes before readings were made. The voltage was then measured for a range of currents that corresponded to temperatures from 1200°K down to 450°K. Readings on the current and voltage from 600°K down to room temperature were made with the lamp mounted in the open room and also mounted in an oil bath which was surrounded by an ice bath. When the lamp was mounted in the oil bath about 30 cm of the bulb projected above the oil and about 50 cm was immersed in the oil. The results under these two conditions of operation for this temperature range agreed very well. Calculations show that at 660°K there should be about 0.5 percent difference in the input whether the lamp was surrounded by an ice bath or at room temperature, which in this case was 20°C. The experimental results agreed with this calculation. The resistance as measured with the lamp in the ice bath came out too high in comparison with the room temperature values which was disturbing until this was recognized as a case of reverse end loss, that is, since the base of the lamp was not inserted in the ice bath, heat was being conducted into the filament from the base, which was at room temperature, thus heating the filament slightly and giving too high value for the resistance. When measurements were being made at these low temperatures liquid air was kept on the side tube containing the charcoal so as to keep the vacuum as low as possible, since the assumption is made that there are no losses from the lower end of this filament except the radiation losses. To see whether the end losses affected the characteristics of the central 25 cm of this filament at low temperatures an attempt was made to eliminate entirely these end losses. While most of the lamp was inserted in the ice bath the voltage and current were measured with the base of the lamp surrounded by a furnace which was so operated that the base was at approximately the same temperature as the lower end of the filament. With the base of the lamp and the lower part of the filament at the same temperature, there should be no conduction of heat either to or from the lower part of the filament. By this method readings were made for a temperature range between about 470 and 319°K. This experiment showed that the calculations based on Dr. Langmuir's equations were correct in that 40 cm of tungsten 0.02 cm in diameter as used for this filament was sufficient to eliminate end losses for a temperature of 450°K.

After all these measurements were made on this lamp it was broken open and dimensions of the filament determined. The total amount of filament between the two potential leads was very carefully weighed and then its length determined by measuring the length of its trace on a piece of blue-print paper with a measuring wheel. This gave a first determination of the diameter of the wire, with 19.31 for the density of tungsten. A straight piece of this same filament was very carefully weighed and its length directly measured thus permitting a check on the diameter or the length previously obtained. This method of determining the diameter gives an average that partially corrects for the slight out-of-round found for the wire. To check these values another lamp was made with samples of the same wire, the difference in this case being that the bulb contained two filaments of different lengths. Except that this lamp had no attached charcoal tube, it was treated in every respect like the first, and measurements were made for a temperature range of 1500 to 2800°K. The final results agreed with the average of those from the first lamp to a little better than one-half percent in specific resistance.

Another test on the temperature-resistance relation of tungsten wire was made with some very pure 25-mil (0.064 cm in diameter) tungsten wire that was manufactured in 1926. This filament was about 60 cm in length with potential leads across the lower 25 cm. It was first made up in an atmosphere of nitrogen-argon mixture and aged for more than an hour at 3000°K and then evacuated and the test made on the filament in a vacuum. The resistance was measured for three temperatures between 1600 and 2200°K after which the lamp was broken open and the dimensions of the wire measured. These values check the results on the first lamp to about 0.2 percent in resistance.

Since all three of these tests included a determination of the physical dimensions of the filament along with the temperature and resistance, the checks are thought to be very satisfactory particularly since the results on these three lamps were checked relatively at high temperatures by measurements of the temperature and electrical characteristics of a number of lamps made of the same wire.

The logarithm of the specific resistance was plotted against the logarithm of the temperature for the entire temperature range covered by these tests and the curve was found to be linear from 1200°K to 2800°K with a slope of 1.195, with only a slight change in the slope somewhere between 1200°K and room temperature. A straight line drawn through the log temperature-log resistance for 1200°K and 293°K as experimentally determined has a slope of 1.209. The slope probably does not change abruptly at a temperature of about 1200°K but sufficient data were not available to determine the shape of the curve at the low temperature region.

Dr. Worthing, while working in this Laboratory, calibrated a number of regular 40-watt tungsten lamps for the temperature-resistance relation for a temperature range from room temperature up to about 400°C by measuring the filament resistance with the lamps in a furnace held at definite temperatures which were measured with a calibrated thermocouple. The results on ten lamps for this range agree very well and give an average slope of 1.20 for the log temperature-log resistance relation.

While the slope of the log temperature-log resistance relation agrees with the previous results[1] the actual values of the resistance for a definite temperature are not in agreement, being about six percent lower and agreeing much better with like values of Jones[2] and of Geiss.[2] We are at a loss to account for the difference between the values of specific resistance as found for these filaments at this time as compared to the previously published results. Part of the change might be due to an actual change in the specific resistance of the tungsten wire as it is manufactured. The ratio of the resistance of a straight filament lamp at room temperature to the resistance of the lamp at its operating condition had been measured for very many lamps previous to the publication of the former results. One reason for a belief in the accuracy of these previous results was that they were in very exact agreement with the ratio of hot to cold resistance of these various tungsten lamps when this ratio was corrected for the effects of end conduction. It is to be noted that the value obtained for the specific resistance of tungsten at room temperature agrees with the value given by Jones[2] to about 0.2 percent.

The data and calculations for the previously[1] published results have been gone over and no error can be located. The agreement of the results on the total emissivity obtained in this test with previous results make it all the more difficult to understand this difference, unless there has been a change in the specific resistance of tungsten wire at high temperatures, since the specific resistance-temperature relation and the total emissivity-temperature relation depend

TABLE I. *Characteristics of tungsten wire.*[1]

Temperature (degrees K) T	Resistivity (microohms cm) ρ[2]	$\frac{T}{\rho}\frac{d\rho}{dT}$	Normal brightness (candles per cm^2) B_n	$\frac{T}{B}\frac{dB}{dT}$	Total emissivity e_t	Total radiation intensity (watts/cm^2) n[3]	$\frac{T}{n}\frac{dn}{dT}$
273	5.05	1.209			0.022	0.00070	
293	5.50	1.209			.023	.00095	
300	5.65	1.209			.024	.00110	
400	8.00	1.209			.034	.00495	
500	10.48	1.209			.042	.0150	
600	13.07	1.209			.052	.0385	
700	15.75	1.209			.062	.0850	
800	18.51	1.209			.074	.1730	5.47
900	12.35	1.209			.089	.333	5.50
1000	24.26	1.209			.105	.600	5.51
1100	27.23	1.209			.121	1.01	5.53
1200	30.26	1.195			.138	1.63	5.53
1300	33.29	1.195			.156	2.54	5.52
1400	36.37	1.195			.174	3.82	5.48
1500	39.50	1.195			.192	5.54	5.41
1600	42.67	1.195	0.94	15.2	.207	7.74	5.31
1700	45.88	1.195	2.30	14.4	.222	10.58	5.18
1800	49.12	1.195	5.15	13.7	.236	14.15	5.02
1900	52.40	1.195	10.40	13.0	.248	18.45	4.89
2000	55.71	1.195	20.00	12.3	.259	23.65	4.80
2100	59.05	1.195	35.9	11.7	.269	29.85	4.73
2200	62.42	1.195	61.0	11.2	.278	37.20	4.68
2300	65.82	1.195	100.1	10.8	.286	45.70	4.64
2400	69.25	1.195	156.0	10.3	.294	55.70	4.61
2500	72.71	1.195	234.0	9.9	.301	67.2	4.59
2600	76.20	1.195	345.	9.6	.309	80.6	4.57
2700	79.71	1.195	495.	9.2	.315	95.6	4.56
2800	83.25	1.195	690.	8.9	.321	112.5	4.55
2900	86.81	1.195	950.	8.6	.329	132.5	4.54
3000	90.40	1.195	1270.	8.3	.334	154.5	4.53

[1] Data for well aged, very pure tungsten wire.
[2] These values depend on the dimensions at room temperature.
[3] Surrounded by a black body at 0°K.

upon the same measurement of current, voltage and physical dimensions of the filament.

By using the data from which the specific resistance-temperature relation was determined, a redetermination was made of the total emissivity of tungsten for the temperature range between 2800°K and room temperature. The temperature of the filament was measured with an optical pyrometer for temperatures from 2800°K down to about 1200°K and temperatures between this and room temperatures were read from a straight line log temperature-log resistance plot between 1200°K and room temperature. As has already been pointed out a straight line very nearly represented the log temperatures. It was only necessary to increase the slope from 1.195 to 1.209 at about 1200°K in order to have it pass through the room temperature resistance point. The values for different temperature of the specific resistance, the total emissivity and some other data on tungsten read from a smooth curve drawn as nearly as possible through the different experimental points are shown in Table I. The values of the total emissivity of tungsten given in Table I agree with the previously published values to better than 0.5 percent of their values over the range from 1200°K to 2800°K. By using known values of the spectral emissivity[6] for the visible and infrared spectrum, the total emissivity was calculated for 1000°, 600°, 400° and 273°K for the filament surrounded by a black body at 0°K. This calculation shows that for a temperature of 600°K the total emission would be changed by only about four percent whether the surroundings were at 0°K

[6] A. G. Worthing, Phys. Rev. **10**, 377 (1917); W. Weniger and A. H. Pfund, ibid. **14**, 427 (1919).

or at 273°K and that the difference would be less than one percent whether it were surrounded by a black body at 293°K or at 273°K. These calculated values agreed with the experimental results to within the accuracy of measurement.

It is difficult to estimate the accuracy of true temperature measurements of a metallic surface. In the case of tungsten the heat treatment above described has always been considered to give the tungsten wire a good polish of a definite nature. This is borne out by the agreement of our temperature resistance relations obtained during many years. If the higher temperatures were corrected so that a straight line log-temperature log-resistance curve would pass through all the points from room temperature to higher temperature the high temperature would have to be corrected by about 25°C. It is not thought that any such error exists in the true temperature scale of properly aged tungsten filaments.

The brightness of tungsten is related to that of a black body by the following relation:

$$\log B_w = \log B_{bb} + \frac{C_2 \log e}{\lambda}\left(\frac{1}{T_c} - \frac{1}{S}\right).$$

B_w is the brightness of tungsten at a color temperature of T_c corresponding to a brightness temperature of S for wave-length λ; B_{bb}, the brightness of a black body at a temperature of T_c. The part of the equation at the right of the plus sign is the logarithm of what has been called the color emissivity of tungsten. This equation would give the exact brightness of tungsten if the color match meant an exact spectral match but since it has been shown[7] that this is not the case a slight correction has to be made for this difference. The correction due to a color match not giving the same spectral distribution for tungsten is about 0.7 percent at 2600°K and about 0.3 percent at 1600°K. With this equation together with these corrections and the brightness of a black body as published by Ives[8] the brightness of tungsten has been calculated and is shown in Table I. These values agree to better than a percent with the previous results which were the average of brightness obtained by this method and by two other methods involving direct measurements.

[7] W. E. Forsythe, J. O. S. A. and R. S. I. **7**, 1115 (1923).
[8] Herbert E. Ives, J. O. S. A. and R. S. I. **12**, 75 (1926).

Journal of the Optical Society of America
The Review of Scientific Instruments

STATEMENT OF CASH RECEIPTS AND DISBURSEMENTS*

FOR THE PERIOD FROM JANUARY 1, 1933 TO DECEMBER 31, 1933

Cash on Hand, January 1, 1933:		
Checking Account	$ 107.52	
Interest Accounts	2,294.26	$2,401.78
Receipts:		
In account with Optical Society of America:		
Sale of back numbers (Volumes 1–5)	$ 37.40	
Advertising collected for O.S.A.	71.00	
Payment from Treasurer (for 1932)	1,122.00	$1,230.40
In account with Scientific Apparatus Makers of America:		
Advertising		$ 858.50
Miscellaneous receipts:		
Sale of back numbers (Volumes 6–)	$ 71.48	
Dartmouth College—Special subvention	600.00	
Reimbursement	.10	
American Institute of Physics, Special cash fund	391.86	
Interest on interest accounts	71.82	$1,135.26
Total		$5,625.94
Disbursements:		
Printer	$1,834.12	
Office expenses	61.43	
American Institute of Physics	401.50	
Miscellaneous	39.93	
American Institute of Physics, Payments against Special cash fund	306.70	$2,643.68
Cash on Hand, December 31, 1933:		
Checking Account	$ 616.18	
Interest Accounts	2,366.08	$2,982.26
Total		$5,625.94

The above is a correct statement of accounts to the best of my knowledge and belief.

(*Signed*) F. K. RICHTMYER, *Editor*.

We, the undersigned, duly appointed to audit the accounts of the *Journal of the Optical Society of America* and *The Review of Scientific Instruments* at the close of business, December 31, 1933, hereby certify that we have examined the statement of receipts, and disbursements, the vouchers, check book and bank book and find them to agree with each other. We, therefore, certify that the above statement is a true statement of the accounts of the *Journal of the Optical Society of America* and *The Review of Scientific Instruments* as of December 31, 1933.

(*Signed*) G. E. GRANTHAM
F. R. HIRSH, JR.

* Cash transactions occurring in 1933 mainly as a result of business done in 1932. (Operation of these journals in 1933 was delegated to the American Institute of Physics, Incorporated.)